

PRINT Your Name: _____

Quiz 4 — September 11, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.

Circle your answer. **Show your work.**

The quiz is worth 5 points.

The function $J_0(x)$ is defined by

$$J_0(x) = \frac{1}{\pi} \int_0^\pi \cos(x \sin t) dt.$$

Find a function f and an interval $[a, b]$ for which $J_0(1)$ is the average value of f over $[a, b]$.

Answer: We know that the average of $f(x)$ on $[a, b]$ is $\frac{1}{b-a} \int_a^b f(x) dx$. We also know that the average of $f(t)$ on $[a, b]$ is $\frac{1}{b-a} \int_a^b f(t) dt$. The problem tells us that $J_0(1) = \frac{1}{\pi} \int_0^\pi \cos(\sin t) dt$. Our job is to find $f(t)$ and $[a, b]$ so that

$$\frac{1}{\pi} \int_0^\pi \cos(\sin t) dt = \frac{1}{b-a} \int_a^b f(t) dt.$$

We take

$$\boxed{f(t) = \cos(\sin t) \quad \text{and} \quad [a, b] = [0, \pi].}$$