PRINT Your Name:____________________

Quiz 3 — September 12, 2012 – Section 10 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Circle your answer. **Show your work.** Your work should be correct and coherent. **CHECK** your answer.
The quiz is worth 5 points.

Find $\int \sqrt{x^2+2x}dx$.

Answer: We complete the square: $x^2 + 2x = (x^2 + 2x + 1) - 1 = (x + 1)^2 - 1$. Let $x + 1 = \sec\theta$. It follows that $(x + 1)^2 - 1 = \tan^2\theta$ and $dx = \sec\theta\tan\theta d\theta$. The original problem is equal to

$$\int \tan^2\theta \sec\theta d\theta.$$

We use integration by parts. Let $u = \tan\theta$ and $dv = \sec\theta\tan\theta d\theta$. It follows that $du = \sec^2\theta d\theta$ and $v = \sec\theta$. So

$$\int \tan^2\theta\sec\theta d\theta = \sec\theta\tan\theta - \int \sec^3\theta d\theta = \sec\theta\tan\theta - \int (\tan^2\theta + 1)\sec\theta d\theta$$

$$= \sec\theta\tan\theta - \int \sec\theta d\theta - \int \tan^2\theta\sec\theta d\theta.$$

Add $\int \tan^2\theta\sec\theta d\theta$ to both sides to see that

$$2\int \tan^2\theta\sec\theta d\theta = \sec\theta\tan\theta - \int \sec\theta d\theta.$$

So

$$\int \sqrt{x^2+2x}dx = \int \tan^2\theta\sec\theta d\theta = (1/2)\left[\sec\theta\tan\theta - \int \sec\theta d\theta\right]$$

$$= (1/2)\left[\sec\theta\tan\theta - \ln|\sec\theta + \tan\theta|\right] + C$$

$$= \boxed{(1/2)\left[(x+1)\sqrt{x^2+2x} - \ln|(x+1) + \sqrt{x^2+2x}|\right] + C}.$$

Check. The derivative of

$$(1/2)\left[(x+1)\sqrt{x^2+2x} - \ln[(x+1) + \sqrt{x^2+2x}\right]$$

is

$$
\begin{aligned}
&(1/2)\left[\frac{(x+1)(2x+2)}{2\sqrt{x^2+2x}}+\sqrt{x^2+2x}-\frac{1+\frac{2x+2}{2\sqrt{x^2+2x}}}{(x+1)+\sqrt{x^2+2x}}\right]\\
&=(1/2)\left[\frac{(x+1)^2}{\sqrt{x^2+2x}}+\sqrt{x^2+2x}-\frac{1+\frac{x+1}{\sqrt{x^2+2x}}}{(x+1)+\sqrt{x^2+2x}}\right]\\
&=(1/2)\left[\frac{(x+1)^2}{\sqrt{x^2+2x}}+\sqrt{x^2+2x}-\frac{\sqrt{x^2+2x}+x+1}{\left[(x+1)+\sqrt{x^2+2x}\right]\sqrt{x^2+2x}}\right]\\
&=(1/2)\left[\frac{(x+1)^2}{\sqrt{x^2+2x}}+\sqrt{x^2+2x}-\frac{1}{\sqrt{x^2+2x}}\right]\\
&=\frac{1}{2\sqrt{x^2+2x}}\left[(x+1)^2+x^2+2x-1\right]\\
&=\frac{1}{2\sqrt{x^2+2x}}\left[2x^2+4x\right]\\
&=\frac{1}{\sqrt{x^2+2x}}\left[x^2+2x\right]=\sqrt{x^2+2x}.\ \checkmark
\end{aligned}
$$