PRINT Your Name:_____

Remove everything from your desk except this page and a pencil or pen.
The solution will be posted soon after the quiz is given.

$\boxed{\text{Circle}}$ your answer. **Show your work. Your work must be correct and coherent. Check your answer.**

The quiz is worth 5 points.

Find $\int \frac{10}{(x-1)(x^2+9)} dx$.

Answer: We apply the technique of partial fractions. We must find constants A, B, C so that

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}.$$

Multiply both sides by $(x-1)(x^2+9)$. We must solve

$$10 = A(x^2+9) + (Bx+C)(x-1)$$

$$10 = \begin{cases} Ax^2 & & +A9 \\ +Bx^2 & -Bx & \\ & +Cx & -C \end{cases}$$

We must find constants A, B, C so that

$$10 = (A+B)x^2 + (C-B)x + (9A-C).$$

Equate the corresponding coefficients:

$$0 = A+B$$
$$0 = C-B$$
$$10 = 9A-C$$

The top equation gives $B = -A$. The middle equation gives $C = B$; so $C = -A$. The bottom equation gives $10 = 9A - C$ and this is $10 = 10A$. We conclude that $A = 1$, $B = -1$, and $C = -1$. Thus, we have shown that

$$\frac{10}{(x-1)(x^2+9)} = \frac{1}{x-1} + \frac{-x-1}{x^2+9}.$$

By the way, this is correct. The right hand side is

$$\frac{(x^2+9)+(x-1)(-x-1)}{(x-1)(x^2-9)} = \frac{(x^2+9)-x^2-x+x+1}{(x-1)(x^2-9)} = \frac{10}{(x-1)(x^2+9)}.$$

We now have

$$\int \frac{10}{(x-1)(x^2+9)} dx = \frac{1}{x-1} + \frac{-x-1}{x^2+9}$$

$$= \boxed{\ln|x-1| - \frac{1}{2}\ln(x^2+9) - \frac{1}{3}\arctan\frac{x}{3} + C}.$$

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