PRINT Your Name:_____

Quiz 26 — November 17, 2015

Approximate $\sum_{k=1}^{\infty} \frac{1}{k^2}$ with an error at most 1/1000. Explain your work.

We see that

$$\left|\sum_{k=1}^{\infty} \frac{1}{k^2} - \sum_{k=1}^{N} \frac{1}{k^2}\right| = \sum_{k=N+1}^{\infty} \frac{1}{k^2} = \text{the area inside the boxes}$$

$$\leq \text{the area under the curve} = \int_N^{\infty} \frac{1}{x^2} dx = \lim_{b\to\infty} \frac{-1}{x}\Big|_N^{\infty} = \lim_{b\to\infty} \frac{-1}{b} + \frac{1}{N} = \frac{1}{N}.$$

We want the error to be at most 1/1000; so we make $\frac{1}{N} \leq 1/1000$. In other words we make $1000 \leq N$. We conclude

$$\boxed{\sum_{k=1}^{1000} \frac{1}{k^2} \text{ approximates } \sum_{k=1}^{\infty} \frac{1}{k^2} \text{ with an error at most } 1/1000.}$$