PRINT Your Name:____________________

Quiz 24 — November 11, 2015

Does $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ converge? Justify your answer.

We apply the integral test to the function $f(x) = \frac{1}{x \ln x}$. We see that $0 < f(x)$ for $2 \le x$. We also see that $f'(x) = -\frac{x\frac{1}{x} + \ln x}{(x \ln x)^2} < 0$ for $2 \le x$. So $f(x)$ is a positive decreasing function for $2 \le x$. Thus $\int_2^{\infty} \frac{1}{x \ln x} dx$ and $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ both converge or both diverge.

We compute

$$\int_2^{\infty} \frac{1}{x \ln x} dx = \lim_{b \to \infty} \ln(\ln x)\Big|_2^b = \lim_{b \to \infty} \ln(\ln b) - \ln(\ln 2) = \infty.$$

The integral diverges; so

$$\boxed{\text{The series } \sum_{n=2}^{\infty} \frac{1}{n \ln n} \text{ also diverges.}}$$