PRINT Your Name:____________________

Quiz 2 — September 9, 2012 – Section 2 – 4:40 – 5:30

Remove everything from your desk except this page and a pencil or pen. The solution will be posted soon after the quiz is given.

Circle your answer. **Show your work. Your work must be correct and coherent. Check your answer.**
The quiz is worth 5 points.

Find $\int \frac{x}{\sqrt{x^2+x+1}}dx$.

Answer: Complete the square in the denominator:

$$x^2+x+1 = x^2+x+(1/4)+(3/4) = (x+1/2)^2+(3/4).$$

Let $x+(1/2) = \sqrt{3}/2\tan\theta$. It follows that $dx = \sqrt{3}/2\sec^2\theta d\theta$,

$$\sqrt{(x+(1/2))^2+(3/4)} = \sqrt{(3/4)\tan^2\theta+(3/4)} = \sqrt{(3/4)(\tan^2\theta+1)}$$

$$= \sqrt{(3/4)(\sec^2\theta)} = (\sqrt{3}/2)\sec\theta,$$

and $x = (\sqrt{3}/2)\tan\theta-(1/2)$. The integral is equal to

$$\int \frac{(\sqrt{3}/2)\tan\theta-(1/2)}{(\sqrt{3}/2)\sec\theta}\sqrt{3}/2\sec^2\theta d\theta = \int((\sqrt{3}/2)\tan\theta-(1/2))\sec\theta d\theta$$

$$= (1/2)\int(\sqrt{3}\tan\theta\sec\theta-\sec\theta)d\theta = (1/2)[\sqrt{3}\sec\theta-\ln|\sec\theta+\tan\theta|]+C$$

$$= (1/2)[2\sqrt{x^2+x+1}-\ln|(2/\sqrt{3})\sqrt{x^2+x+1}+(2/\sqrt{3})(x+(1/2))|]+C$$

$$= (1/2)[2\sqrt{x^2+x+1}-\ln(2/\sqrt{3})-\ln|\sqrt{x^2+x+1}+x+(1/2)|]+C$$

$$= (1/2)[2\sqrt{x^2+x+1}-\ln|\sqrt{x^2+x+1}+x+(1/2)|]+K \quad \text{for } K = -(1/2)\ln(2/\sqrt{3}).$$

$$= \boxed{\sqrt{x^2+x+1}-(1/2)\ln|\sqrt{x^2+x+1}+x+(1/2)|+K}$$

Check: The derivative of the proposed answer is

$$\frac{2x+1}{2\sqrt{x^2+x+1}} - \frac{\frac{2x+1}{2\sqrt{x^2+x+1}}+1}{2(\sqrt{x^2+x+1}+x+(1/2))}$$

$$
\begin{aligned}
&= \frac{2x+1}{2\sqrt{x^2+x+1}} - \frac{2x+1+2\sqrt{x^2+x+1}}{2\sqrt{x^2+x+1}(2\sqrt{x^2+x+1}+2x+1)} \\
&= \frac{2x+1}{2\sqrt{x^2+x+1}} - \frac{1}{2\sqrt{x^2+x+1}} \\
&= \frac{2x+1-1}{2\sqrt{x^2+x+1}} \\
&= \frac{2x}{2\sqrt{x^2+x+1}} \\
&= \frac{x}{\sqrt{x^2+x+1}} \checkmark
\end{aligned}
$$