PRINT Your Name:____________________

Quiz 2 — September 9, 2012 – Section 1 – 3:30 – 4:20

Remove everything from your desk except this page and a pencil or pen.
The solution will be posted soon after the quiz is given.

$\boxed{\text{Circle}}$ your answer. **Show your work. Your work must be correct and coherent. Check your answer.**
The quiz is worth 5 points.

Find $\int \sqrt{5+4x-x^2}dx$.

Answer: Complete the square:

$$5+4x-x^2 = -(x^2-4x)+5 = -(x^2-4x+4)+5+4 = 9-(x-2)^2.$$

Let $x-2 = 3\sin\theta$. It follows that $dx = 3\cos\theta d\theta$ and

$$\sqrt{5+4x-x^2} = \sqrt{9-(x-2)^2} = \sqrt{9-9\sin^2\theta} = 3\sqrt{1-\sin^2\theta} = 3\cos\theta.$$

The integral equals

$$9\int \cos^2\theta d\theta = (9/2)\int(1+\cos(2\theta))d\theta = (9/2)(\theta + \tfrac{\sin(2\theta)}{2}) + C$$

$$= (9/2)(\theta + \tfrac{2\sin\theta\cos\theta}{2}) + C = (9/2)(\theta+\sin\theta\cos\theta) + C$$

$$= (9/2)(\arcsin(\tfrac{x-2}{3}) + \tfrac{(x-2)}{3}\tfrac{\sqrt{9-(x-2)^2}}{3}) + C$$

$$\boxed{= (9/2)(\arcsin(\tfrac{x-2}{3}) + \tfrac{(x-2)}{3}\tfrac{\sqrt{5+4x-x^2}}{3}) + C}$$

(To figure out the value of $\cos\theta$, I drew a right triangle with opposite side of length $x-2$ and hypothenuse side of length 3. It follows that the adjacent side has length $\sqrt{9-(x-2)^2} = \sqrt{5+4x-x^2}$; hence $\cos\theta$ is $\frac{\sqrt{5+4x-x^2}}{3}$.)

Check: The derivative of the proposed answer is

$$\frac{9}{2}\left[\frac{\frac{1}{3}}{\sqrt{1-(\frac{x-2}{3})^2}} + \frac{1}{9}\left[\frac{(x-2)(4-2x)}{2\sqrt{5+4x-x^2}} + \sqrt{5+4x-x^2}\right]\right]$$

$$= \frac{9}{2}\left[\frac{3(\frac{1}{3})}{3\sqrt{1-(\frac{x-2}{3})^2}} + \frac{1}{9}\left[\frac{2(x-2)(2-x)}{2\sqrt{5+4x-x^2}} + \sqrt{5+4x-x^2}\right]\right]$$

$$= \frac{1}{2}\left[\frac{9}{\sqrt{5+4x-x^2}} + \frac{-x^2+4x-4}{\sqrt{5+4x-x^2}} + \sqrt{5+4x-x^2}\right]$$

$$= \frac{1}{2\sqrt{5+4x-x^2}}\left[9 - x^2 + 4x - 4 + 5 + 4x - x^2\right]$$

$$= \frac{1}{2\sqrt{5+4x-x^2}}[2(5+4x-x^2)] = \sqrt{5+4x-x^2}. \checkmark$$