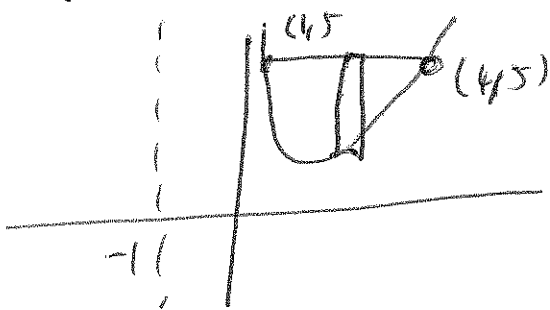



Quiz Tuesday Oct 20 #18

Revolve the region bounded by $y=5$ and $y=x+\frac{4}{x}$ about $x=-1$. Find the volume of the resulting solid.



We must use shells.

I do not want to solve $y=x+\frac{4}{x}$ for x . Drop rectangles parallel to $x=-1$. Spin the rectangle.

Get a shell  of volume $2\pi rh t$ where

$$t=dx \quad r=x+1 \quad h=5-x-\frac{4}{x}$$

The intersection occurs when $5=x+\frac{4}{x}$ so $5x=x^2+4$

$$\text{so } x^2-5x+4=0 \quad \text{or } (x-4)(x-1)=0$$

The volume of a shell is $2\pi rh t = 2\pi (x+1)(5-x-\frac{4}{x}) dx$.

The volume of the solid is

$$2\pi \int_1^4 (x+1)(5-x-\frac{4}{x}) dx = 2\pi \int_1^4 (5x+5-x^2-x-4-\frac{4}{x}) dx$$

$$= 2\pi \int_1^4 (4x+1-x^2-\frac{4}{x}) dx = 2\pi \left(2x^2+x-\frac{x^3}{3}-4\ln|x| \right) \Big|_1^4$$

$$= 2\pi \left(32+4-\frac{64}{3}-4\ln 4 - \left(2+1-\frac{1}{3} \right) \right)$$

$$= 2\pi \left(33 - \frac{63}{3} - 4\ln 4 \right) = 2\pi (12 - 4\ln 4)$$