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**Quiz – April 4, 2006**

The series

$$1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots$$

satisfies the hypotheses of the Alternating Series Test; and therefore this series converges. Approximate the sum of this series with an error at most .005. **Explain very thoroughly.**

**Answer:** Let  $S$  be the sum of the series. The Alternating Series test tells us that the distance between  $S$  and some partial sum of the series is at most the absolute value of the next term:

$$\begin{aligned} |S - 1| &\leq \frac{1}{3!} \\ |S - (1 - \frac{1}{3!})| &\leq \frac{1}{5!} \\ |S - (1 - \frac{1}{3!} + \frac{1}{5!})| &\leq \frac{1}{7!} \\ &\text{etc.} \end{aligned}$$

We want to find an odd number  $n$  with

$$\frac{1}{n!} \leq .005 = \frac{5}{1000} = \frac{1}{200}.$$

We want  $200 \leq n!$ . We know that  $5! = 120$  and  $7!$  is much more than 200. We conclude that

$1 - \frac{1}{3!} + \frac{1}{5!}$ approximates $S$ with an error at most .005.
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