The series
\[ 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \ldots \]
satisfies the hypotheses of the Alternating Series Test; and therefore this series converges. Approximate the sum of this series with an error at most .005. **Explain very thoroughly.**

**Answer:** Let \( S \) be the sum of the series. The Alternating Series test tells us that the distance between \( S \) and some partial sum of the series is at most the absolute value of the next term:

\[
|S - 1| \leq \frac{1}{3!}, \\
|S - (1 - \frac{1}{3!})| \leq \frac{1}{5!}, \\
|S - (1 - \frac{1}{3!} + \frac{1}{5!})| \leq \frac{1}{7!}
\]

etc.

We want to find an odd number \( n \) with

\[
\frac{1}{n!} \leq .005 = \frac{5}{1000} = \frac{1}{200}.
\]

We want \( 200 \leq n! \). We know that \( 5! = 120 \) and \( 7! \) is much more than \( 200 \). We conclude that

\[
1 - \frac{1}{3!} + \frac{1}{5!} \text{ approximates } S \text{ with an error at most } .005.
\]