Quiz – March 30, 2006

Does the series \( \sum_{k=1}^{\infty} \frac{5^k + k}{k! + 3} \) converge? **Explain very thoroughly.**

**Answer:** Notice that
\[
\frac{5^k + k}{k! + 3} < \frac{5^k + 5^k}{k!} = \frac{2 \cdot 5^k}{k!}
\]
because the fraction on the right has a larger numerator and a smaller denominator.

The series \( \sum_{k=1}^{\infty} \frac{2 \cdot 5^k}{k!} \) converges by the ratio test:
\[
\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lim_{k \to \infty} \frac{\frac{2 \cdot 5^{k+1}}{(k+1)!}}{\frac{2 \cdot 5^k}{k!}} = \lim_{k \to \infty} \frac{2 \cdot 5^k}{(k + 1)!} \cdot \frac{k!}{5^k} = \lim_{k \to \infty} \frac{5}{k + 1} = 0 < 1.
\]

Both series \( \sum_{k=1}^{\infty} \frac{5^k + k}{k! + 3} \) and \( \sum_{k=1}^{\infty} \frac{2 \cdot 5^k}{k!} \) are positive series. The terms of \( \sum_{k=1}^{\infty} \frac{5^k + k}{k! + 3} \) are smaller than the terms of \( \sum_{k=1}^{\infty} \frac{2 \cdot 5^k}{k!} \). The series \( \sum_{k=1}^{\infty} \frac{2 \cdot 5^k}{k!} \) converges. We apply the Comparison test to conclude that the series \( \sum_{k=1}^{\infty} \frac{5^k + k}{k! + 3} \) also converges.