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**Quiz – February 23, 2006**

Find $\int \frac{dx}{x^2\sqrt{9x^2-4}}$ . **Check your answer.**

**Answer:** Let $3x = 2\sec\theta$ . It follows that $3dx = 2\sec\theta\tan\theta d\theta$ ,

$$\sqrt{9x^2-4} = \sqrt{4\sec^2\theta - 4} = 2\tan\theta,$$

and the original integral is equal to

$$\int \frac{(2/3)\sec\theta\tan\theta d\theta}{(4/9)\sec^2\theta(2)\tan\theta} = \frac{3}{4}\int \frac{d\theta}{\sec\theta} = \frac{3}{4}\int \cos\theta d\theta = \frac{3}{4}\sin\theta + C.$$

Consider a triangle with hypotenuse $3x$ , adjacent $2$ , and opposite $\sqrt{9x^2-4}$ . We see that the answer is

$$\frac{3}{4}\frac{\sqrt{9x^2-4}}{3x} + C = \boxed{\frac{\sqrt{9x^2-4}}{4x} + C.}$$

**Check:** The derivative of the proposed answer is

$$\frac{4x\frac{18x}{2\sqrt{9x^2-4}} - 4\sqrt{9x^2-4}}{16x^2} = \frac{\frac{9x^2}{\sqrt{9x^2-4}} - \sqrt{9x^2-4}}{4x^2} = \frac{9x^2-(9x^2-4)}{4x^2\sqrt{9x^2-4}}. \checkmark$$