PRINT Your Name:__________________

Quiz 13 — November 28, 2012 – – Section 9 – 10:10 – 11:00

Remove everything from your desk except a pencil or pen.

Write in complete sentences.
The quiz is worth 5 points.

Consider the power series $f(x) = \sum_{n=1}^{\infty} \frac{3^n(x+4)^n}{\sqrt{n}}$. Where does $f(x)$ converge? **Justify your answer very thoroughly. Write in complete sentences.** If there are "endpoints" in your answer, be sure to study the endpoints very carefully.

Answer. We know that $f(x)$ converges when $x = -4$. Henceforth we only consider x with $x \neq -4$. Apply the ratio test. Let

$$\rho = \lim_{n\to\infty} \frac{\frac{3^{n+1}|x+4|^{n+1}}{\sqrt{n+1}}}{\frac{3^n|x+4|^n}{\sqrt{n}}} = \lim_{n\to\infty} \frac{3^{n+1}|x+4|^{n+1}}{\sqrt{n+1}} \frac{\sqrt{n}}{3^n|x+4|^n} = \lim_{n\to\infty} 3|x+4|\frac{\sqrt{n}}{\sqrt{n+1}}$$

$$= \lim_{n\to\infty} 3|x+4|\frac{1}{\sqrt{1+\frac{1}{n}}} = 3|x+4|.$$

If $3|x+4| < 1$ then $f(x)$ converges. If $1 < 3|x+4|$ diverges. We notice that $3|x+4| < 1$ is equivalent to $\frac{-13}{3} < x < \frac{-11}{3}$. We also notice that $1 < 3|x+4|$ is equivalent to $x < \frac{-13}{3}$ or $\frac{-11}{3} < x$. Now we deal with the end points. We see that

$$f(\tfrac{-11}{3}) = \sum_{n=1}^{\infty} \frac{3^n(\frac{-11}{3}+4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n(\frac{1}{3})^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}},$$

which is the p-series with $p = \frac{1}{2} < 1$. This p-series diverges; hence, $f(\frac{-11}{3})$ diverges. We also see that

$$f(\tfrac{-13}{3}) = \sum_{n=1}^{\infty} \frac{3^n(\frac{-13}{3}+4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n(\frac{-1}{3})^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}},$$

which is an alternating series. We observe that

$$\frac{1}{\sqrt{1}} > \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}} > \dots$$

and $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$. We apply the Alternating Series Test and conclude that $f(\frac{-13}{3})$ converges. The final conclusion is that

$$\boxed{f(x) \text{ converges for } \frac{-13}{3} \leq x < \frac{-11}{3} \text{ and } f(x) \text{ diverges everywhere else.}}$$