PRINT Your Name:____________________

**Quiz 12 — November 14, 2012 – – Section 9 – 10:10 – 11:00**

**Remove everything from your desk except a pencil or pen.**

**Write in complete sentences.**
The quiz is worth 5 points.

Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n}$ converge? If so, approximate the sum of this series with an error less than $10^{-4}$. **Justify your answer very thoroughly. Write in complete sentences.**

**Answer.** We apply the alternating series test. This series is an alternating series. It is clear that

$$\frac{1}{1 \cdot 5^1} > \frac{1}{2 \cdot 5^2} > \frac{1}{3 \cdot 5^3} > \frac{1}{4 \cdot 5^4} > \dots .$$

It is also clear that $\lim_{n\to\infty} \frac{1}{n5^n} = 0$. The alternating series test guarantees that the series converges. The AST also guarantees that

$$\left| \sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n} - \sum_{n=1}^{N} \frac{(-1)^n}{n5^n} \right| \le \frac{1}{(N+1)5^{N+1}}$$

for each positive integer $N$. We want to choose $N$ large enough that

$$\frac{1}{(N+1)5^{N+1}} \le 10^{-4}.$$

In other words, we want

$$10^4 \le (N+1)5^{N+1}.$$

We see that $4 \cdot 5^4 = 4 \cdot 625 < 10^4 < 5 \cdot 5 \cdot 625 = 5 \cdot 5^5$. So $10^4 \le (N+1)5^{N+1}$ when $N = 4$. We conclude that

> the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n}$ converges and the sum of the series may be approximated by $\sum_{n=1}^{4} \frac{(-1)^n}{n5^n}$ with an error less than $10^{-4}$.