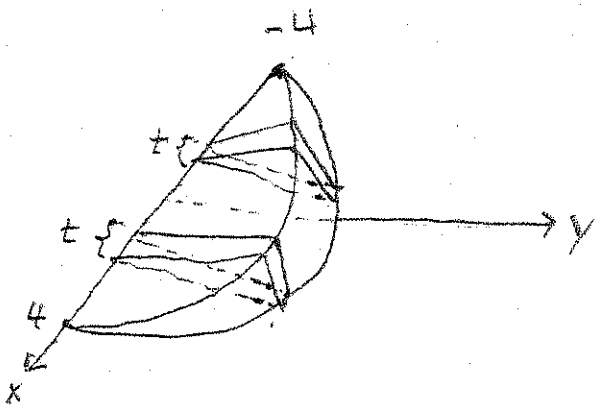
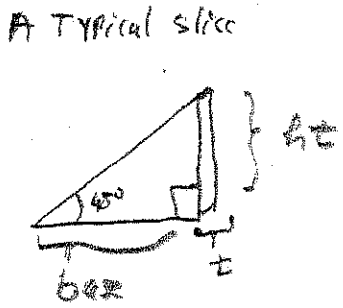
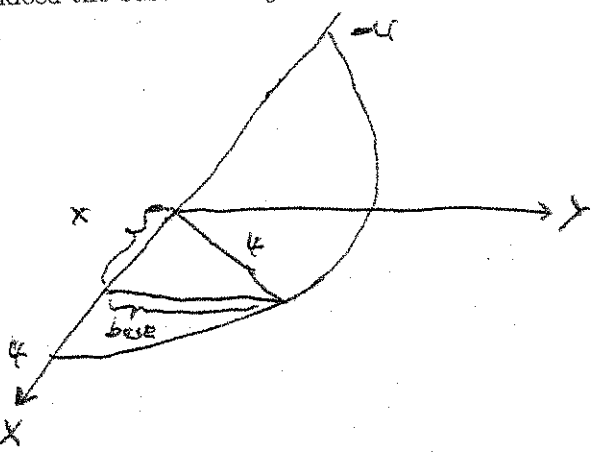
2. (6 points) **Consider a wedge cut from a cylinder of radius 4. This wedge is cut using 2 planes. The first plane is perpendicular to the axis of the cylinder. The second plane intersects the first plane through a diameter of the cylinder. The angle of intersection of the two planes is 45 degrees. Find the volume of the wedge.**

We view the first plane as the xy-plane. We make the intersection of the two planes occur on the x-axis from $x = -4$ to $x = 4$. We slice the wedge into a collection of triangular slices by partitioning the x-axis from $x = -4$ to $x = 4$. Each slice has thickness $t = dx$. Each slice has a base and a height. The volume of a given slice is $\frac{1}{2}$ base times height times thickness. The base and the height are equal (because the angle of intersection of the two planes is 45 degrees).





The base depends on which slice we are studying. In other words, the base depends on the x-coordinate of our slice. Indeed the base is the y-coordinate on the circle with center $(0,0)$ and radius 4.



So the base is $\sqrt{16-x^2}$. The volume of the slice with x-coordinate x is $(1/2)$ base times height times thickness, which equals $(1/2)\sqrt{16-x^2}\sqrt{16-x^2}dx = (1/2)(16-x^2)dx$. The volume of the solid is

$$\tfrac{1}{2}\int_{-4}^{4}(16-x^2)dx = \tfrac{1}{2}(16(x)-\tfrac{x^3}{3})\Big|_{-4}^{4}$$

$$= 2\tfrac{1}{2}(64-\tfrac{64}{3}) = \tfrac{2}{3}(64) = \boxed{\frac{128}{3}}.$$