PRINT Your Name:__________________

Quiz 12 — November 14, 2012 – Section 10 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Write in complete sentences.
The quiz is worth 5 points.

Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ converge? If so, approximate the sum of this series with an error less than 10^{-4}. **Justify your answer very thoroughly. Write in complete sentences.**

Answer. We apply the alternating series test. This series is an alternating series. It is clear that

$$\frac{1}{1^5} > \frac{1}{2^5} > \frac{1}{3^5} > \frac{1}{4^5} > \dots .$$

It is also clear that $\lim_{n\to\infty} \frac{1}{n^5} = 0$. The alternating series test guarantees that the series converges. The AST also guarantees that

$$\left| \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5} - \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n^5} \right| \le \frac{1}{(N+1)^5}$$

for each positive integer N. We want to choose N large enough that

$$\frac{1}{(N+1)^5} \le 10^{-4}.$$

In other words, we want

$$10^4 \le (N+1)^5.$$

We see that $6^5 < 40 \cdot 40 \cdot 6 = 9600 < 10^4 < 11,200 = 40 \cdot 40 \cdot 7 < 7^5$. So $10^4 \le (N+1)^5$ when $N = 6$. We conclude that

the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ converges and the sum of the series may be approximated by $\sum_{n=1}^{6} \frac{(-1)^{n+1}}{n^5}$ with an error less than 10^{-4}.