Quiz 11 November 3, 2010 – Section 9 – 10:10 – 11:00

Does the series $\sum_{n=1}^{\infty} \frac{n!}{e^{(n^2)}}$ converge? **Justify your answer very thoroughly.**

Answer. We use the ratio test. Let

$$\rho = \lim_{n\to\infty} \frac{\frac{(n+1)!}{e^{((n+1)^2)}}}{\frac{n!}{e^{(n^2)}}} = \lim_{n\to\infty} \frac{(n+1)!}{e^{((n+1)^2)}} \frac{e^{(n^2)}}{n!} = \lim_{n\to\infty} \frac{n+1}{e^{(n^2+2n+1)}} e^{(n^2)} = \lim_{n\to\infty} \frac{n+1}{e^{2n+1}} = 0.$$

(One may use L'Hopital's rule to do the last limit.) Thus, $\rho < 1$. The ratio test tells us that $\sum_{n=1}^{\infty} \frac{n!}{e^{(n^2)}}$ converges.