

PRINT Your Name: _____

Quiz 11 — March 30 — Section 8 — 10:10 — 11:00

Remove everything from your desk except a pencil or pen.

Write in complete sentences.

The quiz is worth 5 points.

Does the series $\sum_{n=1}^{\infty} \frac{n!}{e^{(n^2)}}$ converge? **Justify your answer very thoroughly.**

Answer. We use the ratio test. Let

$$\rho = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)!}{e^{((n+1)^2)}}}{\frac{n!}{e^{(n^2)}}} = \lim_{n \rightarrow \infty} \frac{(n+1)! e^{(n^2)}}{e^{((n+1)^2)} n!} = \lim_{n \rightarrow \infty} \frac{n+1}{e^{(n^2+2n+1)}} e^{(n^2)} = \lim_{n \rightarrow \infty} \frac{n+1}{e^{2n+1}} = 0.$$

(One may use L'Hopital's rule to do the last limit.) Thus, $\rho < 1$. The ratio test tells us that $\sum_{n=1}^{\infty} \frac{n!}{e^{(n^2)}}$ converges.