PRINT Your Name:__________________

Quiz 11 — November 7, 2012 – Section 10 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Write in complete sentences.
The quiz is worth 5 points.

Does the series $\sum_{n=1}^{\infty} \tan \frac{1}{n}$ converge? **Justify your answer very thoroughly.**

Answer. We compare this series to the divergent Harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$, using the limit comparison test. We know that the top and the bottom of the following limit both go to 0, so we use L'hopital's rule to see that

$$\lim_{n\to\infty} \frac{\tan \frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{-1}{n^2} \sec^2 \frac{1}{n}}{\frac{-1}{n^2}} = \lim_{n\to\infty} \sec^2 \frac{1}{n} = 1.$$

We know that 1 is a number, not zero, not infinity. So, $\sum_{n=1}^{\infty} \tan \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ both converge or both diverge. We know that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. We conclude that $\sum_{n=1}^{\infty} \tan \frac{1}{n}$ diverges.