PRINT Your Name:________________

Quiz 11 — November 11, 2013 – Section 1 – 3:30 – 4:20

Remove everything from your desk except a pencil or pen.

Write in complete sentences. Explain your work!
The quiz is worth 5 points.

Estimate the sum $\sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!}$ with an error at most .000005. **Explain what you are doing VERY THOROUGHLY. Write in complete sentences.**

Answer: We apply the alternating series test. The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!}$ is an alternating series. The terms (in absolute value) $\frac{1}{10^n n!}$ are decreasing and go to zero. (Indeed, the numerators are constant and the denominators are growing and become arbitrarily large.). So the Alternating Series Test applies. Thus the series converges and the distance between any given partial sum and the sum of the whole series is at most the absolute value of the necxt term; that is:

$$\left| \sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!} - \sum_{n=0}^{N} \frac{(-1)^n}{10^n n!} \right| \le \frac{1}{10^{N+1}(N+1)!}.$$

We want to pick N large enough that $\frac{1}{10^{N+1}(N+1)!} \le 5(10)^{-6}$. We want

$$10^6 \le 5(10)^{N+1}(N+1)!.$$

Observe that when $N = 3$, then

$$10^6 < 1.2(10^6) = 5(10)^4(24) = 5(10)^{N+1}(N+1)!.$$

We conclude that

$$\boxed{\sum_{n=0}^{3} \frac{(-1)^n}{10^n n!} \text{ approximates } \sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!} \text{ with an error of at most } 10^6.}$$