Quiz 10 October 27, 2010 – Section 9 – 10:10 – 11:00

Does the series $\sum_{n=1}^{\infty} \sin \frac{1}{n}$ converge? **Justify your answer very thoroughly.**

Answer. We compare this series to the divergent Harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$, using the limit comparison test. We know that the top and the bottom of the following limit both go to 0, so we use L'hopital's rule to see that

$$\lim_{n\to\infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{-1}{n^2} \cos \frac{1}{n}}{\frac{-1}{n^2}} = \lim_{n\to\infty} \cos \frac{1}{n} = 1.$$

We know that 1 is a number, not zero, not infinity. So, $\sum_{n=1}^{\infty} \sin \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ both converge or both diverge. We know that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. We conclude that $\sum_{n=1}^{\infty} \sin \frac{1}{n}$ diverges.