PRINT Your Name:

Quiz 10 — October 31, 2012 – Section 10 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Circle your answer. Show your work. Your work should be correct and coherent.

The quiz is worth 5 points.

Does the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ converge? Justify your answer. Write in complete sentences.

Answer: Yes, the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2 + 1}$ does converge. We compare $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2 + 1}$ to $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Both series are series of non-negative numbers. We know that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is the *p*-series with p = 2 > 1; hence the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. Furthermore, $\frac{\cos^2 n}{n^2 + 1} < \frac{1}{n^2}$ because $\cos^2 n \le 1$ and $n^2 \le n^2 + 1$ so

$$\frac{\cos^2 n}{n^2 + 1} \le \frac{1}{n^2 + 1} < \frac{1}{n^2}.$$

Our conclusion follows from the comparison test.