

PRINT Your Name: _____

Quiz 10 — September 17, 2015

Remove everything from your desk except this page and a pencil or pen.

The solution will be posted soon after the quiz is given.

Circle your answer. **Show your work.** Your work must be correct and coherent. **Check your answer.**

Find $\int \frac{5x^2+3x-2}{x^3+2x^2} dx$.

Answer: Notice that $x^3 + 2x^2 = x^2(x + 2)$. We use the technique of partial fractions. We look for numbers A and B with

$$\frac{5x^2 + 3x - 2}{x^3 + 2x^2} = \frac{A}{x + 2} + \frac{B}{x} + \frac{C}{x^2}.$$

Multiply both sides by $x^2(x + 2)$ to obtain

$$5x^2 + 3x - 2 = Ax^2 + Bx(x + 2) + C(x + 2)$$

$$5x^2 + 3x - 2 = (A + B)x^2 + (2B + C)x + 2C.$$

Equate the corresponding coefficients.

$$\begin{aligned} 5 &= A + B \\ 3 &= 2B + C \\ -2 &= 2C \end{aligned}$$

So, $C = -1$, $B = 2$, and $A = 3$. We seem to have shown that

$$\frac{5x^2 + 3x - 2}{x^3 + 2x^2} = \frac{3}{x + 2} + \frac{2}{x} + \frac{-1}{x^2}.$$

Before going any further, we verify this assertion. The right side is

$$\frac{3x^2 + 2x(x + 2) - (x + 2)}{x^2(x + 2)} = \frac{5x^2 + 3x - 2}{x^3 + 2x^2},$$

as desired. Now we compute

$$\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx = \int \left(\frac{3}{x + 2} + \frac{2}{x} + \frac{-1}{x^2} \right) dx = \boxed{3 \ln |x + 2| + 2 \ln |x| + \frac{1}{x} + C}.$$