

PRINT Your Name: _____

Quiz 10 — October 30, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.

Circle your answer. **Show your work.**

The quiz is worth 5 points.

Compute

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots .$$

Answer: The sum is equal to

$$\lim_{n \rightarrow \infty} \left[\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1) \cdot (2n+1)} \right].$$

We do the technique of partial fractions to examine

$$\frac{1}{a \cdot (a+2)} = \frac{A}{a} + \frac{B}{a+2}.$$

Multiply both sides by $a \cdot (a+2)$ to get

$$1 = A(a+2) + Ba.$$

Set $a = -2$ to see $-1/2 = B$. Set $a = 0$ to see that $1/2 = A$. Do check that:

$$\frac{1}{2} \left(\frac{1}{a} - \frac{1}{a+2} \right) = \frac{1}{2} \frac{a+2-a}{a(a+2)} = \frac{1}{a(a+2)}.$$

So the original sum is

$$\lim_{n \rightarrow \infty} \frac{1}{2} \left[\left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{7} \right) + \cdots + \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \right].$$

Remove the interior parentheses. Every term (except the first and last terms) cancels with the term next to it. The original sum is

$$= \lim_{n \rightarrow \infty} 1/2 \left[\frac{1}{1} - \frac{1}{2n+1} \right] = 1/2.$$