Quiz for August 21, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.

$\boxed{\text{Circle}}$ your answer. **Show your work.**
The quiz is worth 5 points.

Compute $\int_0^1 \frac{xdx}{\sqrt{4-3x^4}}$.

Answer: We plan to maneuver the given integral into the form

$$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C.$$

We have

$$\int_0^1 \frac{xdx}{\sqrt{4-3x^4}} = \int_0^1 \frac{xdx}{\sqrt{4\left(1-\frac{3x^4}{4}\right)}} = \int_0^1 \frac{xdx}{2\sqrt{1-\frac{3x^4}{4}}}.$$

Now we are ready to substitute. Let $u = \frac{\sqrt{3}x^2}{2}$. It follows that $du = \sqrt{3}xdx$. Notice that when $x = 0$, then $u = 0$; and when $x = 1$, then $u = \frac{\sqrt{3}}{2}$. The original problem is equal to

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{du}{2\sqrt{3}\sqrt{1-u^2}} = \frac{1}{2\sqrt{3}} \arcsin u \Big|_0^{\frac{\sqrt{3}}{2}} = \frac{1}{2\sqrt{3}}\left(\arcsin\frac{\sqrt{3}}{2} - \arcsin 0\right)$$

$$= \frac{1}{2\sqrt{3}}\left(\frac{\pi}{3} - 0\right) = \boxed{\frac{\pi}{6\sqrt{3}}}.$$