Final 142 Fall 1998

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PRINT Your Name: _____

There are 19 problems on 8 pages. Problems 1 and 2 are worth 7 points each. Each of the other problems is worth 8 points. SHOW your work. CIRCLE your answer. **Check** your answer whenever posible. **No Calculators.**

1. Find the Taylor polynomial of degree three, $P_3(x)$, for $f(x) = \ln x$ about $a = 1$.

$f(x) = \ln(x)$ $\quad f(1) = 0$

$f'(x) = \frac{1}{x}$ $\quad f'(1) = 1$

$f''(x) = -\frac{1}{x^2}$ $\quad f''(1) = -1$

$f'''(x) = +\frac{2}{x^3}$ $\quad f'''(1) = 2$

$f^{(4)}(x) = \frac{-6}{x^4}$

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$$P_3(x) = f(1) + f'(1)(x-1) + f''(1)\frac{(x-1)^2}{2!} + f'''(1)\frac{(x-1)^3}{3!}$$

$$P_3(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$$

2. Find an upper bound for the difference between $f(x)$ and $P_3(x)$ (from problem 1) when $|x-1| \le \frac{1}{10}$.

$$|f(x) - P_3(x)| = |R_3(x)| = \left|\frac{f^{(4)}(c)}{4!}(x-1)^4\right| = \frac{|-6|}{c^4\,4!}|x-1|^4 \le \frac{6}{(.9)^4 10^4\,4!}$$

$.9 < c < 1.1$

$\frac{1}{1.1} < \frac{1}{c} < \frac{1}{.9}$

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$$|f(x) - P_3(x)| \le \frac{6}{(.9)^4 10^4\,4!}$$