5. Does the series \( \sum_{n=1}^{\infty} \frac{2n-1}{n^3+1} \) converge or diverge? Justify your answer.

\[
\frac{2n-1}{n^3+1} < \frac{2}{n^2} \quad \text{because} \quad 2n^3-n^2 < 2n^3+2
\]

\[\sum \frac{2}{n^2} = 2 \sum \frac{1}{n^2} \quad \text{which converges. It is a}\]

\[\sum \frac{2n-1}{n^3+1} \quad \text{by the comparison test}\]

6. Where does the function \( f(x) = \sum_{n=1}^{\infty} (x-7)^n \) converge?

Use the ratio test.

Let \( p = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(x-7)^{n+1}}{(x-7)^n} \right| = \lim_{n \to \infty} |x-7| = |x-7| \)

\( |x-7| < 1 \) the series converges

\( |x-7| > 1 \) the series diverges

\( f(8) = \sum_{n=1}^{\infty} \frac{1}{2^n} \) which diverges (by \( \frac{1}{n} \) term test)

\( f(6) = \sum_{n=1}^{\infty} \frac{1}{1^n} \) which diverges

\( f(x) \) converges for \( 6 < x < 8 \) and diverges elsewhere.