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5. Does the series $\sum_{n=1}^{\infty} \frac{2n-1}{n^3+1}$ converge or diverge? Justify your answer.

$\frac{2n-1}{n^3+1} < \frac{2}{n^2}$ because $2n^3 - n^2 < 2n^3 + 2$

$\sum \frac{2}{n^2} = 2 \sum \frac{1}{n^2}$ which converges. It is a p-series with $1 < 2 = p$

$\therefore \sum_{n=1}^{\infty} \frac{2n-1}{n^3+1}$ converges by the comparison test

6. Where does the function $f(x) = \sum_{n=1}^{\infty} (x-7)^n$ converge?

Use the ratio Test.

Let $\rho = \lim_{n \to \infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n \to \infty} \left|\frac{(x-7)^{n+1}}{(x-7)^n}\right| = \lim_{n \to \infty} |x-7| = |x-7|$

$|x-7| < 1$ then the series converges

If $|x-7| > 1$ then the series diverges

8 $f(8) = \sum_{n=1}^{\infty} (1)^n$ which diverges (by n^{th} term test)

6 $f(6) = \sum_{n=1}^{\infty} (-1)^n$ " " " " " "

$f(x)$ converges for $6 < x < 8$ and diverges elsewhere.