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3. Find $\int \frac{\ln x}{x^3} dx = \frac{-\ln x}{2x^2} + \frac{1}{2} \int x^{-3} dx = \frac{-\ln x}{2x^2} - \frac{1}{4x^2} + C$

$u = \ln x \quad v = \frac{x^{-2}}{2}$
 $du = \frac{1}{x} dx \quad dv = x^{-3} dx$

check

$$\frac{d}{dx} \left(\frac{-\ln x}{2x^2} - \frac{1}{4x^2} \right) = -\ln x (-x^{-3}) - \frac{1}{2x^3} + \frac{1}{2x^3} = \frac{\ln x}{x^3} \checkmark$$

4. Does the series $\sum_{n=1}^{\infty} \frac{n}{e^n}$ converge or diverge? Justify your answer.

Ratio Test Let $\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{n+1}{e^{n+1}} \frac{e^n}{n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \frac{1}{e} = \frac{1}{e} < 1$

Thus the series converges