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9. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at 400°F and left to cool in a room at 75°F , then its temperature T after t hours will satisfy the differential equation

$$\frac{dT}{dt} = k(T - 75).$$

If the temperature fell to 200°F after one hour, what will it be after 3 hours?

$$T(0) = 400$$

$$T(1) = 200$$

$$\therefore \frac{125}{325} = e^k$$

Find $T(3)$

$$\int \frac{dT}{T-75} = \int k dt$$

$$\therefore \ln \frac{5}{13} = k$$

$$\ln |T-75| = kt + C$$

$$|T-75| = e^{kt+C}$$

$$T-75 = (\pm e^C) e^{kt}$$

$$\text{Let } \pm e^C = K$$

$$T-75 = K e^{kt}$$

$$-75 = T(0) - 75 = K$$

$$325 = K$$

$$25 = 200 - 75 = T(1) - 75 = 325 e^k$$

$$\therefore T(t) = 75 + 325 e^{\ln(\frac{5}{13})t}$$

$$T(3) = 75 + 325 e^{\ln(\frac{5}{13})3}$$

10. Find $\int \cos 4x \cos 3x dx$.

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$$

$$\frac{1}{2} (\cos(A+B) + \cos(A-B)) = \cos A \cos B$$

$$\text{Take } A = 4x \quad B = 3x$$

$$\text{Problem} = \frac{1}{2} \int \cos 7x + \cos x dx$$

$$= \frac{1}{2} \left(\frac{\sin 7x}{7} + \sin x \right) + C$$