7. Where does $f(x) \sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}$ converge? Where does $f(x)$ diverge? Justify your answer.

Use the ratio Test. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x|^{2n+3}}{(2n+3)!}\frac{(2n+1)!}{|x|^{2n+1}}$

$= \lim_{n\to\infty}\frac{|x|^2}{(2n+2)(2n+3)} = 0 < 1$

$\therefore f(x)$ converges for all X.

8. Where does $f(x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{2^n n(n+2)}$ converge? Where does $f(x)$ diverge? Justify your answer.

Ratio test $\rho = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{|x-1|^{n+1}}{2^{n+1}(n+1)(n+3)}\frac{2^n n(n+2)}{|x-1|^n}$

$= \lim_{n\to\infty}\frac{|x-1|}{2}\frac{1(1+\frac{2}{n})}{(1+\frac{1}{n})(1+\frac{3}{n})} = \frac{|x-1|}{2}$

If $\left|\frac{x-1}{2}\right| < 1$, then $f(x)$ converges

If $-2 < x-1 < 2$, the $f(x)$ converges

If $-1 < x < 3$, then $f(x)$ converges

If $x < -1$ or $3 < x$, the $1 < \frac{|x-1|}{2}$ and $f(x)$ diverges

at $x=-1$ $f(-1) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(-2)^n}{2^n n(n+2)} = -\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$

compare $\sum\frac{1}{n(n+2)}$ to $\sum\frac{1}{n^2}$ (which converges because it is the p-series with $p=2>1$)

$\frac{1}{n(n+2)} < \frac{1}{n^2}$

so a straight comparison shows $\sum\frac{1}{n(n+2)}$ converges,

So $f(-1) = -\sum\frac{1}{n(n+2)}$ converges

$f(3) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n(n+2)}$ we just saw that $\sum\frac{1}{n(n+2)}$ converges.

so $f(3)$ converges by the Absolute Convergence Test

So $f(x)$ converges for $-1 \le x \le 3$ and $f(x)$ diverges everywhere else.