PRINT Your Name:____________________
There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. [CIRCLE] your answer. NO CALCULATORS! If you want to pick up your exam before Monday, write a short note to that effect on the top of this page and I will leave your exam outside my office door, before I go home tonight.

1. Does \( \sum_{n=1}^{\infty} \frac{4}{n} \) converge? Justify your answer.

This series is 4 times the harmonic series. [This series diverges.]

2. Does \( \sum_{n=1}^{\infty} \frac{n+3}{n^2 \sqrt{n}} \) converge? Justify your answer.

Do a limit comparison to \( \sum_{n=1}^{\infty} \frac{1}{n^{10}} \), which converges, because it is a

\[
\lim_{n \to \infty} \frac{\frac{n+3}{n^2 \sqrt{n}}}{\frac{1}{n^{10}}} = \lim_{n \to \infty} \frac{n+3}{n^{11/2}} = \lim_{n \to \infty} \frac{n+3}{n^{5.5}} = \lim_{n \to \infty} \frac{1 + \frac{3}{n}}{n^{5/2}} = 0 \]

\( a_n \sim \frac{n+3}{n^2 \sqrt{n}} \) \( b_n = \frac{1}{n^{10}} \)

It is not \( 0 < a_n < b_n \), so both series converge or both series diverge. We know that \( \sum_{n=1}^{\infty} \frac{1}{n^{5/2}} \) converges, so \( \frac{\sum_{n=1}^{\infty} \frac{n+3}{n^2 \sqrt{n}}}{\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}} \) also converges.