Fall 2001, Exam 3, Math 142

PRINT Your Name: ______________________

There are 11 problems on 6 pages. Problem 1 is worth 10 points. Each of the
other problems is worth 9 points. SHOW your work. [CIRCLE] your answer.

NO CALCULATORS! CHECK your answer whenever possible. If you want
to pick up your exam before Monday, write a short note to that effect on the top
of this page and I will leave your exam outside my office door, before I go home

tonight.

1. Find \( \int \sin^3 x \, dx \). Check your answer.

2. Find \( \int \cos^4 x \, dx \).

3. Find the general solution of \( \frac{dy}{dx} - 3y = xe^{3x} \). Check your answer.

4. Find \( \int e^{-x} \cos x \, dx \). Check your answer.

5. Find \( \int \frac{6x^2 - 3x + 1}{(4x + 1)(x^2 + 1)} \, dx \). Check your answer.

6. Consider the sequence whose \( n^{th} \) term is \( a_n = \left( \frac{n-3}{n} \right)^n \). Find the limit of this
sequence.

7. Find \( \lim_{x \to 1^-} \frac{x-1}{\arctan x} \).

8. Find \( \int_{-3}^1 \frac{1}{x^2} \, dx \).

9. Find \( \int_1^\infty \frac{x}{e^x} \, dx \).

10. Consider the series \( \sum_{k=1}^{\infty} \ln\left( \frac{k}{k+1} \right) \). Find a closed formula for the partial sum
\( s_n = \sum_{k=1}^{n} \ln\left( \frac{k}{k+1} \right) \). (In other words, I want you to find a formula which is equal
to \( s_n \). Your formula is not allowed to contain any “dots” or any summation
signs.) Does the original series converge or diverge? Find the limit of the series,
if possible.

11. A ball is dropped from a height of 100 feet. Each time it hits the floor, it
rebounds to \( \frac{2}{3} \) its previous height. Find the total distance the ball travels
before coming to rest.