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17. Does $\sum_{n=3}^{\infty}\left[\frac{3}{n}-\frac{3}{n+1}\right]$ converge? (Explain your answer.)

The m^{th} partial sum is $\sum_{n=3}^{m} \frac{3}{n} - \frac{3}{n+1} = \left(\frac{3}{3}-\cancel{\frac{3}{4}}\right)+\left(\cancel{\frac{3}{4}}-\cancel{\frac{3}{5}}\right)+\left(\cancel{\frac{3}{5}}-\cancel{\frac{3}{6}}\right)+\cdots+\left(\cancel{\frac{3}{m}}-\frac{3}{m+1}\right)$

$= 1 - \frac{3}{m+1}$

The sum of the series is the limit of the sequence of partial sums

$= \lim_{m\to\infty} 1 - \frac{3}{m+1} = 1$

This series $\sum_{n=3}^{\infty}\left(\frac{3}{n}-\frac{3}{n+1}\right)$ converges to 1.

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18. Does $\sum_{n=2}^{\infty}\frac{8}{n\ln n}$ converge? (Explain your answer.)

I use the integral test $f(x) = \frac{8}{x\ln x}$ is positive and decreasing

$\int_2^{\infty} \frac{8}{x\ln x}dx = \lim_{b\to\infty} 8\ln(\ln x)\Big]_2^b = \lim_{b\to\infty} 8\ln(\ln b) - 8\ln(\ln 2) = \infty$

The integral diverges so the series diverges.

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