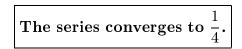
Math 142, Fall 2000, Exam 4, Problems 9, 10, and 11

9. Consider the series $\sum_{k=4}^{\infty} \frac{1}{k} - \frac{1}{k+1}$. Give a closed formula for the partial sum $s_n = \sum_{k=4}^n \frac{1}{k} - \frac{1}{k+1}$. Does the series converge? If so, what is the sum of the series? We see that

$$s_n = \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) + \dots + \left(\frac{1}{n-1} - \frac{1}{n}\right) + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$
$$= \left(\frac{1}{4} - \frac{1}{n+1}\right).$$

We also see that $\lim_{n \to \infty} s_n = \lim_{n \to \infty} \left(\frac{1}{4} - \frac{1}{n+1} \right) = \frac{1}{4}$. Thus,



10. Find
$$\int_0^\infty \frac{dx}{1+x^2}$$
.
We have

$$\int_0^\infty \frac{dx}{1+x^2} = \lim_{b \to \infty} \int_0^b \frac{dx}{1+x^2} = \lim_{b \to \infty} \tan^{-1} x \Big|_0^b = \lim_{b \to \infty} \tan^{-1} b - \tan^{-1}(0)$$
$$= \frac{\pi}{2} - 0 = \boxed{\frac{\pi}{2}}.$$

11. Find $\lim_{x \to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4}$.

Use L'hopitals' rule four times to get that the limit is $\frac{1}{24}$; OR use

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

Thus,

$$\lim_{x \to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4} = \lim_{x \to 0} \frac{\left(1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right) - 1 + \frac{x^2}{2}}{x^4}$$
$$= \lim_{x \to 0} \frac{\frac{x^4}{4!} - \frac{x^6}{6!} + \dots}{x^4} = \lim_{x \to 0} \frac{1}{4!} - \frac{x^2}{6!} + \dots = \boxed{\frac{1}{4!}}.$$