9. Find the third Taylor polynomial $P_3(x)$ for $f(x) = \ln(x + 1)$ about $a = 0$.

\[
\begin{align*}
(f(x)) &= x, \quad f(0) = 0 \\
(f'(x)) &= \frac{1}{x+1}, \quad f'(0) = 1 \\
(f''(x)) &= -\frac{1}{(x+1)^2}, \quad f''(0) = -1 \\
(f'''(x)) &= \frac{2}{(x+1)^3}, \quad f'''(0) = 2 \\
(f''''(x)) &= \frac{-6}{(x+1)^5}
\end{align*}
\]

\[
P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3
\]

\[
P_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}
\]

10. Take $P_3(x)$ and $f(x)$ from problem 9. Find an upper bound for the error that is introduced if $f(x)$ is approximated by $P_3(x)$ for $|x| \leq \frac{1}{100}$.

\[
|P_3(x) - f(x)| = |R_3(x)| = \left| \frac{f^{(4)}(c)}{4!} x^4 \right| = \left| \frac{-6}{(1+c)^4} \frac{x^4}{4!} \right| 
\]

\[
\leq \frac{1}{(0.99)^4 (100)^4}
\]

\[
\frac{1}{1+c} \leq x \leq 0.01
\]

$c$ is between $x$ and $0$

\[
|\frac{1}{1+c}| \leq |1-0.01| = \frac{1}{99}
\]