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9. Find the third Taylor polynomial $P_3(x)$ for $f(x) = \ln(x+1)$ about $a = 0$.

$f(x) = \ln(x+1) \quad f(0) = 0$

$f'(x) = \frac{1}{x+1} \quad f'(0) = 1$

$f''(x) = \frac{-1}{(x+1)^2} \quad f''(0) = -1$

$f'''(x) = \frac{2}{(x+1)^3} \quad f'''(0) = 2$

$f''''(x) = \frac{-6}{(x+1)^4}$

$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3$

$P_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$

10. Take $P_3(x)$ and $f(x)$ from problem 9. Find an upper bound for the error that is introduced if $f(x)$ is approximated by $P_3(x)$ for $|x| \le \frac{1}{100}$.

$|P_3(x) - f(x)| = |R_3(x)| = \left|\frac{f^{(4)}(c)}{4!}x^4\right| = \left|\frac{-6x^4}{(1+c)^4 4!}\right| = \frac{(x)^4}{(1+c)^4 4} \le \frac{1}{(.99)^4 4(100)^4}$

$-.01 \le x \le .01$

c is between x and 0

so $-.01 \le c \le .01$

$\frac{1}{1+c}$ is largest when c is smallest

$\therefore \frac{1}{1+c} \le \frac{1}{1-.01} = \frac{1}{.99}$

If $|x| \le \frac{1}{100}$, then

$|P_3(x) - f(x)| \le \frac{1}{(.99)^4 4(100)^4}$