

9. Let $f(x) = \frac{2x+1}{3x-1}$ for $x \neq 1/3$. Find $f^{-1}(x)$.

$x = y = f^{-1}(x)$. Find y

So $f(y) = x$

$$\frac{2y+1}{3y-1} = x$$

$$2y+1 = x(3y-1)$$

$$y(2-3x) = -x-1$$

$$y = \frac{-x-1}{2-3x}$$

$f^{-1}(x) = \frac{x+1}{3x-2} \text{ for } x \neq \frac{2}{3}$

ch $f(f^{-1}(x)) = \frac{2\left(\frac{x+1}{3x-2}\right) + 1}{3\left(\frac{x+1}{3x-2}\right) - 1} = \frac{2x+4 + 3x-2}{3x+3 - 3x+2}$

$$= \frac{5x}{5} = x \checkmark$$

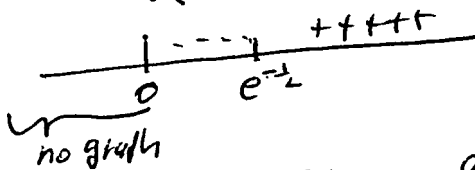
$$f^{-1}(f(x)) = \frac{\frac{2x+1}{3x-1} + 1}{3\left(\frac{2x+1}{3x-1}\right) - 2} = \frac{2x+1 + 3x-1}{6x+3 - 6x+2}$$

$$= \frac{5x}{5} = x \checkmark$$

10. Let $f(x) = x^2 \ln x$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find the local maxima, local minima, and points of inflection of $y = f(x)$. Graph $y = f(x)$.

$$f' = x^2 \cdot \frac{1}{x} + 2x \ln x = x + 2x \ln x$$

$$= x(1 + 2 \ln x)$$



no graph
 $f' = 0$ when $1 + 2 \ln x = 0 \Rightarrow \ln x = -\frac{1}{2}$
 $x = e^{-\frac{1}{2}}$

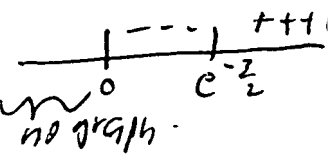
f is \downarrow for $0 < x < e^{-\frac{1}{2}}$
 f is \uparrow for $e^{-\frac{1}{2}} < x$
 $(e^{-\frac{1}{2}}, -\frac{1}{2e})$ is a loc. min

f is \downarrow for $0 < x < e^{-\frac{1}{2}}$
 f is \uparrow for $e^{-\frac{1}{2}} < x$
 $(e^{-\frac{1}{2}}, -\frac{1}{2e})$ is a loc. min
 no loc. max

$$f'' = x\left(\frac{2}{x}\right) + 1 + 2 \ln x$$

$$f'' = 3 + 2 \ln x$$

$$f'' = 0 \text{ when } -\frac{3}{2} = \ln x \text{ so } x = e^{-\frac{3}{2}}$$



$$\lim_{x \rightarrow 0^+} x^2 \ln x = 0$$

$x^2 \ln x \rightarrow 0$ as $x \rightarrow 0^+$
 $\ln x$ has two asymptotes $-\infty$
 $x^2 \ln x$ has so L'Hopital's rule

