Math 142, Fall 2000, Exam 4

PRINT Your Name: ________________________________

There are 11 problems on 6 pages. Problem 1 is worth 10 points. Each of the other
problems is worth 9 points. SHOW your work. [CIRCLE] your answer. NO
CALCULATORS!

1. Find the Taylor polynomial of degree six, \( P_6(x) \), for \( f(x) = \sin x \) about \( a = 0 \).

2. Use your answer to problem 1 to estimate \( \int_0^1 \frac{\sin x}{x} \, dx \). How good is your
estimate? Explain.

3. Use Simpson's rule to estimate the area of the following shape. All
measurements are in feet.

4. Does the series \( \sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^k \) converge? Justify your answer. Find the sum of
the series if you can.

5. Does the series \( \sum_{k=1}^{\infty} \frac{5}{k+3} \) converge? Justify your answer. Find the sum of the
series if you can.

6. Does the series \( \sum_{k=1}^{\infty} \frac{3^{k+1}}{2^{k-1}} \) converge? Justify your answer. Find the sum of the
series if you can.

7. A ball is dropped from the height of 10 feet. Each time it hits the floor it
rebounds to \( \frac{2}{3} \) its previous height. Find the total distance it travels.

8. Consider the series \( \sum_{k=4}^{\infty} \frac{1}{3^k} \). Give a closed formula for the partial sum
\( s_n = \sum_{k=4}^{n} \frac{1}{3^k} \). Does the series converge? If so, what is the sum of the series?

9. Consider the series \( \sum_{k=4}^{\infty} \frac{1}{k} - \frac{1}{k+1} \). Give a closed formula for the partial sum
\( s_n = \sum_{k=4}^{n} \frac{1}{k} - \frac{1}{k+1} \). Does the series converge? If so, what is the sum of the
series?

10. Find \( \int_0^{\infty} \frac{dx}{1 + x^2} \).

11. Find \( \lim_{x \to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4} \).