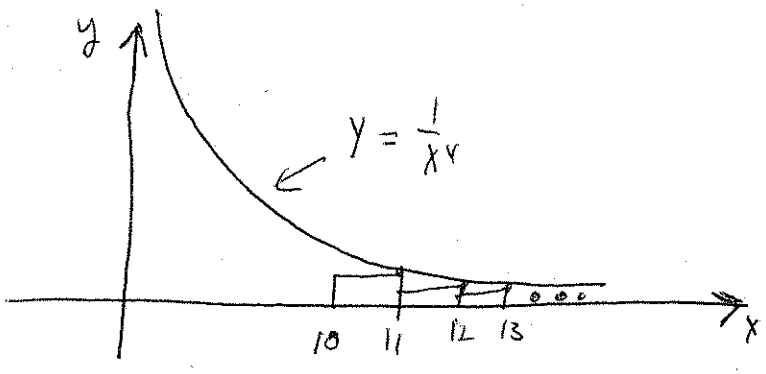
Quiz 10 — March 30, 2011 – Section 4 – 9:05-9:55 recitation..

Estimate the distance between $\sum_{n=2}^{10}(\frac{1}{n^4})$ and $\sum_{n=2}^{\infty}(\frac{1}{n^4})$. I want your estimate to be close to, but larger than the exact distance. **Justify your answer very thoroughly. Use complete sentences.**

Answer. The distance between $\sum_{n=2}^{10}(\frac{1}{n^4})$ and $\sum_{n=2}^{\infty}(\frac{1}{n^4})$ is $\sum_{n=11}^{\infty}(\frac{1}{n^4})$. Look at the picture:



The area inside the boxes is $\sum_{n=11}^{\infty}(\frac{1}{n^4})$. The area inside the boxes is less than the area under the curve. Thus,

$$\sum_{n=11}^{\infty}\left(\frac{1}{n^4}\right) \leq \int_{10}^{\infty}\frac{1}{x^4}dx = \lim_{b\to\infty}\frac{1}{-3x^3}\Big|_{10}^{b} = \lim_{b\to\infty}\frac{1}{-3b^3}+\frac{1}{3000} = \frac{1}{3000}.$$

Thus,

$$\boxed{\sum_{n=2}^{10}\left(\frac{1}{n^4}\right) \text{ approximates } \sum_{n=2}^{\infty}\left(\frac{1}{n^4}\right) \text{ with an error at most } \frac{1}{3000}.}$$