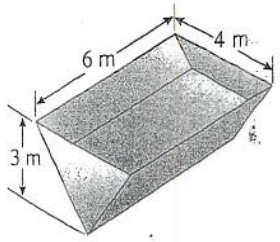
PRINT Your Name:_____

Quiz – September 19, 2006

The vat shown in the accompanying figure contains water to a depth of 2 m. Find the work required to pump all the water to the top of the vat. [Use 9810 N/m^3 as the weight density of water.]



I draw my axis with $y=0$ at the top of the tank. At the beginning the water sits between $y=1$ and $y=3$. I will move this water to $y=0$. Chop the y-axis from $y=1$ to $y=3$ into small pieces. The work required to move the layer of water with y-coordinate y is

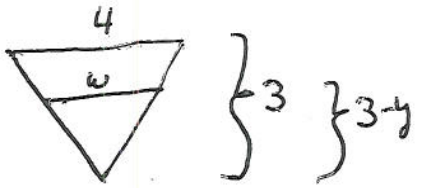
Weight of layer · distance moved = Vol of layer · density · distance

The layer looks like

$t = dy$

$\ell = 6$

Use similar triangles to find w



$$\frac{w}{3-y} = \frac{4}{3} \qquad w = \frac{4}{3}(3-y)$$

(This makes sense. When $y=0$, then $w=4$ and when $y=3$, then $w=0$.)

The work to move the layer of water at y-coordinate y is

$\ell \cdot w \cdot t \cdot \text{density} \cdot y = 6 \cdot \frac{4}{3}(3-y)\, 9810 \cdot y \cdot dy$. Then sum up. Take the limit. Total work $= \sum_1^3 8(9810)(3y-y^2)\,dy$

$$= \left[8(9810)\left(\frac{3y^2}{2} - \frac{y^3}{3}\right) \right]_1^3 \text{ N-m}$$

$= 261{,}600$ Joules