Quiz 7, September 13, 2016

Find $\int \cos^5 x\,dx$. Please check your answer.

Let $u = \sin x$; so $du = \cos x dx$. Observe that

$$\int \cos^5 x\,dx = \int \cos^4 x \cos x\,dx = \int (1-\cos^2 x)^2 \cos x\,dx = \int (1-u^2)^2\,du$$
$$= \int (1-2u^2+u^4)du = u - 2u^3/3 + u^5/5 + C$$
$$= \boxed{\sin x - (2/3)\sin^3 x + (1/5)\sin^5 x + C}.$$

Check: The derivative of the proposed annswer is

$$\cos x - 2\sin^2 \cos x + \sin^4 x \cos x = \cos x(1 - 2\sin^2 x + \sin^4 x) = \cos x(1-\sin^2 x)^2$$
$$= \cos x(\cos^2 x)^2 = \cos^5 x.\checkmark$$