Quiz 5, August 31, 2016

Find $\int e^{2x}\cos 3x dx$.

Answer: Try integration by parts. Let $u = e^{2x}$ and $dv = \cos 3x dx$. We compute $du = 2e^{2x}dx$ and $v = \frac{1}{3}\sin 3x$. So

$$\int e^{2x}\cos 3x dx = \int u dv = uv - \int v du = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x dx.$$

We try integration by parts again with $u = e^{2x}$ and $dv = \sin 3x dx$. We compute $du = 2e^{2x}dx$ and $v = \frac{-1}{3}\cos 3x$. We see that

$$\int e^{2x}\cos 3x dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[\frac{-1}{3}e^{2x}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x dx\right].$$

Add $\frac{4}{9}\int e^{2x}\cos 3x dx$ to both sides to obtain

$$\frac{13}{9}\int e^{2x}\cos 3x dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\frac{(-1)}{3}e^{2x}\cos 3x.$$

Multiply both sides by $\frac{9}{13}$ to see

$$\int e^{2x}\cos 3x dx = \frac{9}{13}\left[\frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\frac{(-1)}{3}e^{2x}\cos 3x\right]$$

$$= \boxed{\frac{e^{2x}}{13}\left[3\sin 3x + 2\cos 3x\right] + C}$$

Check: The derivative of the proposed answer is

$$\frac{e^{2x}}{13}\left[(9\cos 3x - 6\sin 3x) + (6\sin 3x + 4\cos 3x)\right] = e^{2x}\cos 3x \checkmark$$