Quiz 4, August 30, 2016

Find $\int \frac{4t^3-t^2+16t}{t^2+4}dt$.

Answer: First do a long division to write the fraction as a proper fraction. Observe that t^2+4 goes into $4t^3-t^2+16t$, $4t-1$ times with a remainder of 4; hence,

$$\frac{4t^3-t^2+16t}{t^2+4} = 4t-1+\frac{4}{t^2+4}.$$

It is a good idea to check this much before proceeding. The right side is

$$4t-1+\frac{4}{t^2+4} = \frac{(4t-1)(t^2+4)+4}{t^2+4} = \frac{4t^3+16t-t^2-4+4}{t^2+4},$$

and this is the left side. So,

$$\int \tfrac{4t^3-t^2+16t}{t^2+4}dt = \int 4t-1+\tfrac{4}{t^2+4}dt = 2t^2-t+\int \tfrac{4}{t^2+4}dt.$$

There are three ways to find $\int \frac{4}{t^2+4}dt$. There is a formula for $\int \frac{du}{u^2+a^2}$, but I don't know the formula, so I don't use that. One can use "trig substitution". I will teach this in section 8.4. I know and will use $\int \frac{du}{u^2+1} = \arctan u + C$. I wish the 4 in the denominator were a 1; so, I divide top and bottom by 4: Our integral is

$$= 2t^2-t+\int \frac{1}{(t/2)^2+1}dt = 2t^2-t+2\int \frac{1}{u^2+1}du,$$

where $u=t/2$ and $du=dt/2$. Our integral is

$$= 2t^2-t+2\arctan u + C = \boxed{2t^2-t+2\arctan(t/2)+C.}$$

Check. The derivative of the proposed answer is

$$4t-1+2\frac{\frac{1}{2}}{(t/2)^2+1} = 4t-1+\frac{1}{(t/2)^2+1}$$

Multiply top and bottom by 4 to see that the derivative of the proposed answer is

$$4t-1+\frac{4}{t^2+4} = \frac{(4t-1)(t^2+4)+4}{t^2+4} = \frac{4t^3+16t-t^2-4+4}{t^2+4} = \frac{4t^3+16t-t^2}{t^2+4}.\checkmark$$