

Find the area between $x + 4y^2 = 4$ and $x + y^4 = 1$ for $0 \leq x$.

We find the intersection $(4 - 4y^2) + y^4 = 1$

$$\text{So } y^4 - 4y^2 + 3 = 0$$

$$(y^2 - 3)(y^2 - 1) = 0$$

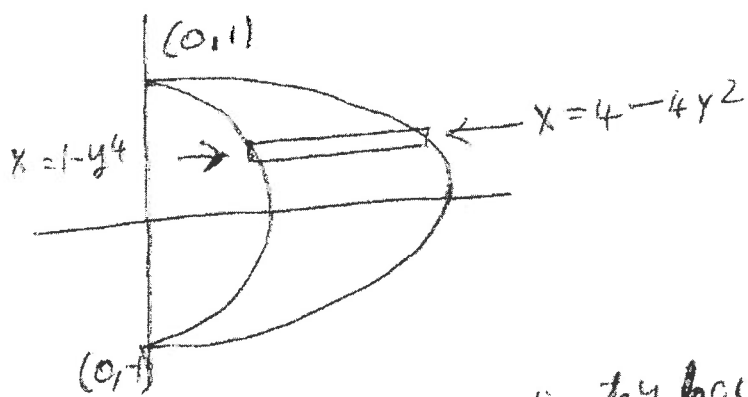
$$y^2 = 3 \text{ or } y^2 = 1$$

When $y^2 = 1$, then $x = 0$

When $y^2 = 3$, then $x = -8$

The two curves have 4 points of intersection: $(0, 1)$, $(0, -1)$, $(-8, \sqrt{3})$ and $(-8, -\sqrt{3})$

Our picture looks like



The rectangle with y -coordinate y has

$$\text{Area } [(4 - 4y^2) - (1 - y^4)] dy$$

The region has area

$$\int_{-1}^1 (3 - 4y^2 + y^4) dy$$

$$= \left[3y - \frac{4y^3}{3} + \frac{y^5}{5} \right]_{-1}^1$$

$$= 2 \left(3 - \frac{4}{3} + \frac{1}{5} \right)$$

$$= \frac{2(45 - 20 + 3)}{15} = \frac{56}{15}$$