Quiz 25, March 29, 2016

Estimate how well $\sum_{n=1}^{4}(-1)^{n+1}\frac{(.01)^n}{n}$ approximates $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n}$. Explain your answer very thoroughly.

Answer: We apply the Alternating Series Test to the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n}$. We see that the series is an alternating series; the numbers

$$\frac{(.01)^1}{1} > \frac{(.01)^2}{2} > \frac{(.01)^3}{3} > \dots$$

are decreasing. (There is no ambiguity in this assertion because $\frac{(.01)^n}{n}$ equals $\frac{1}{(100^n n)}$; when one changes n to $n+1$, the denominator becomes larger, the numerator is unchanged, and the fraction becomes smaller.) The terms go to zero, that is, $\lim_{n\to\infty}\frac{(.01)^n}{n} = 0$. So the Alternating Series Test applies and a partial sum of the series approximates the total sum of the series by at most the absolute value of the next term:

$$\left|\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n} - \sum_{n=1}^{4}(-1)^{n+1}\frac{(.01)^n}{n}\right| \le \frac{(.01)^5}{5} = \frac{1}{5\cdot 10^{10}}.$$

We conclude that

$$\boxed{\sum_{n=1}^{4}(-1)^{n+1}\frac{(.01)^n}{n} \text{ approximates } \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n} \text{ with an error at most } \frac{1}{5\cdot 10^{10}}.}$$