Quiz 25, November 22, 2016

Find the Taylor polynomials of orders 0, 1, 2, and 3 for $f(x) = \sqrt{x}$ about $a = 4$.

Answer: We see that

$$\begin{array}{ll} f(x) = x^{1/2} & f(4) = 2 \\ f'(x) = \frac{1}{2}x^{-1/2} & f'(4) = \frac{1}{4} \\ f''(x) = \frac{-1}{4}x^{-3/2} & f''(4) = \frac{-1}{32} \\ f'''(x) = \frac{3}{8}x^{-5/2} & f'''(4) = \frac{3}{256} \end{array}$$

The Taylor polynomial of order n for $f(x)$ about a is

$$P_n(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + \cdots + f^{(n)}(a)\frac{(x-a)^n}{n!};$$

consequently in our problem

$$\begin{aligned} P_0(x) &= 2 \\ P_1(x) &= 2 + \frac{1}{4}(x-4) \\ P_2(x) &= 2 + \frac{1}{4}(x-4) - \frac{1}{32}\frac{(x-4)^2}{2!} \\ P_3(x) &= 2 + \frac{1}{4}(x-4) - \frac{1}{32}\frac{(x-4)^2}{2!} + \frac{3}{256}\frac{(x-4)^3}{3!} \end{aligned}$$