Quiz 21, November 10, 2016

Estimate how close $\sum_{n=1}^{4}(-1)^{n+1}\frac{(.01)^n}{n}$ is to $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n}$ Justify your answer.

Answer: The Alternating Series Test applies to

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n} = +\frac{1}{100}-\frac{1}{2\times 100^2}+\frac{1}{3\times 100^3}-\frac{1}{4\times 100^4}-\frac{1}{5\times 100^5}+\dots$$

because the series is an alternating series, the terms in absolute value are decreasing, and the terms go to zero. Thus

$$\left|\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n}-\sum_{n=1}^{N}(-1)^{n+1}\frac{(.01)^n}{n}\right| \le \frac{(.01)^{N+1}}{N+1},$$

for all N. For us, $N=4$; thus,

$$\left|\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n}-\sum_{n=1}^{4}(-1)^{n+1}\frac{(.01)^n}{n}\right| \le \frac{(.01)^{5}}{5},$$

and

$$\boxed{\sum_{n=1}^{4}(-1)^{n+1}\frac{(.01)^n}{n} \text{ approximates } \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(.01)^n}{n} \text{ with an error at most } \frac{1}{5\times 100^5}.}$$