Quiz 2, January 19, 2015

Find $\int x^3\sqrt{x^2+1}dx$.

Answer: Let $u = x^2+1$. It follows that $du = 2xdx$ and $u-1 = x^2$. So

$$\int x^3\sqrt{x^2+1}dx = \frac{1}{2}\int(u-1)\sqrt{u}du = \frac{1}{2}\int(u^{3/2}-u^{1/2})du$$

$$= \frac{1}{2}(\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2})+C = \boxed{\frac{1}{2}(\frac{2}{5}(x^2+1)^{5/2}-\frac{2}{3}(x^2+1)^{3/2})+C.}$$

Check: The derivative of the proposed answer is

$$\frac{1}{2}((x^2+1)^{3/2}2x-(x^2+1)^{1/2}2x) = x\sqrt{x^2+1}((x^2+1)-1) = x^3\sqrt{x^2+1} \checkmark$$