Find $\int \sin^5 \left(\frac{x}{3}\right) \cos \left(\frac{x}{3}\right) dx$.

Answer: Let $u = \sin \left(\frac{x}{3}\right)$. Then $du = \left(\frac{1}{3}\right) \cos \left(\frac{x}{3}\right) dx$. It follows that $3du = \cos \left(\frac{x}{3}\right) dx$. The original problem is equal to

$$3 \int u^5 du = 3u^6/6 + C = \boxed{\left(\frac{1}{2}\right) \sin^6 \left(\frac{x}{3}\right) + C}.$$

Check. The derivative of the proposed answer is

$$3 \left(\sin^5 \left(\frac{x}{3}\right) \cos \left(\frac{x}{3}\right)\right) \left(\frac{1}{3}\right) = \sin^5 \left(\frac{x}{3}\right) \cos \left(\frac{x}{3}\right) . \checkmark .$$