

Quiz 13, October 12, 2016

Find $\int \frac{dx}{4x^2 + 8x + 13}$. Please check your answer.

The integral is equal to

$$\int \frac{dx}{4(x^2 + 2x + \boxed{1}) + 13 - 4\boxed{1}} = \int \frac{dx}{(2(x+1))^2 + 9} = \frac{1}{9} \int \frac{dx}{((\frac{2}{3})(x+1))^2 + 1}.$$

Let $u = \frac{2}{3}(x+1)$; so $du = \frac{2}{3}dx$. The integral is equal to

$$\frac{1}{9} \cdot \frac{3}{2} \int \frac{du}{u^2 + 1} = \frac{1}{6} \arctan(u) + C = \boxed{\frac{1}{6} \arctan(\frac{2}{3}(x+1)) + C}.$$

Check: The derivative of the proposed answer is

$$\frac{1}{6} \frac{\frac{2}{3}}{[(\frac{2}{3}(x+1))^2 + 1]} = \frac{1}{9[\frac{4}{9}(x^2 + 2x + 1) + 1]} = \frac{1}{4x^2 + 8x + 13} \checkmark$$