

Quiz 10, February 10, 2016

Find $\int \frac{dx}{1-x^2}$.

Answer: Observe that $\frac{1}{1-x^2} = \frac{-1}{x^2-1} = \frac{-1}{(x-1)(x+1)}$. We use the technique of partial fractions and look for numbers A and B with

$$\frac{-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}.$$

Multiply both sides by $(x-1)(x+1)$ to obtain

$$-1 = A(x+1) + B(x-1)$$

Plug in $x = 1$ to learn that $A = \frac{-1}{2}$. Plug in $x = -1$ to learn that $B = \frac{1}{2}$. We have calculated that

$$\frac{1}{1-x^2} = \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+1}.$$

Before going any further, let's make sure this is correct. We see that the right side is

$$\frac{1}{2} \left[\frac{-(x+1) + (x-1)}{(x-1)(x+1)} \right] = \frac{-1}{x^2-1} = \frac{1}{1-x^2},$$

as desired. We now compute

$$\int \frac{dx}{1-x^2} = \frac{1}{2} \left[\int \frac{-1}{x-1} + \frac{1}{x+1} \right] = \boxed{\frac{1}{2} [-\ln|x-1| + \ln|x+1|] + C}$$