Math 142, Exam 4, Spring 2006

There are 10 problems. Each problem is worth 10 points. Write in complete sentences. JUSTIFY EVERY ANSWER VERY THOROUGHLY.

If I know your e-mail address, I will e-mail your grade to you. If I don’t already know your e-mail address and you want me to know it, then send me an e-mail.

I will post the solutions on my website a few hours after the exam is finished.

1. Does the series \( \sum_{k=1}^{\infty} \frac{1}{2 + \frac{1}{k}} \) converge? Justify your answer.

2. Does the series \( \sum_{k=1}^{\infty} \frac{k}{2^k} \) converge? Justify your answer.

3. Does the series \( \sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}} \) converge? Justify your answer.

4. Does the series \( \sum_{k=1}^{\infty} \frac{\arctan k}{k^2} \) converge? Justify your answer.

5. Does the series \( \sum_{k=2}^{\infty} \frac{1}{\sqrt{k(k-1)}} \) converge? Justify your answer.

6. Consider the sequence \( \{a_n\} \) with \( a_1 = \sqrt{20} \), and \( a_n = \sqrt{20 + a_{n-1}} \) for \( n \geq 2 \). Prove that the sequence \( \{a_n\} \) converges. Find the limit of the sequence \( \{a_n\} \).

7. Approximate the sum \( \sum_{k=1}^{\infty} \frac{1}{k^3} \) with an error of at most \( \frac{1}{100} \). Justify your answer.

8. Approximate the sum \( \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3} \) with an error of at most \( \frac{1}{100} \). Justify your answer.

9. A ball is dropped from the height of 40 feet. Each time it hits the floor it rebounds to \( \frac{3}{4} \) its previous height. Find the total distance it travels. Explain what you are doing.

10. Give a closed formula for \( s_n = \sum_{k=2}^{n} \ln \left(1 - \frac{1}{k^2}\right) \). (Your formula should be exactly equal to the sum I have given. Your formula should not contain any dots or any summation signs.) Explain what you are doing.